



## Sheet 2

- 1] Determine the volume  $V$  of a region defined in a cylindrical coordinate system as  $1\text{ m} \leq r \leq 2\text{ m}$  ,  $0 \leq \phi \leq \frac{\pi}{3}$  rad , and  $0 \leq z \leq 1\text{ m}$  by integration . Check your result without performing the integration.

$$\left[ V = \frac{\pi}{2} \text{ m}^3 \right]$$

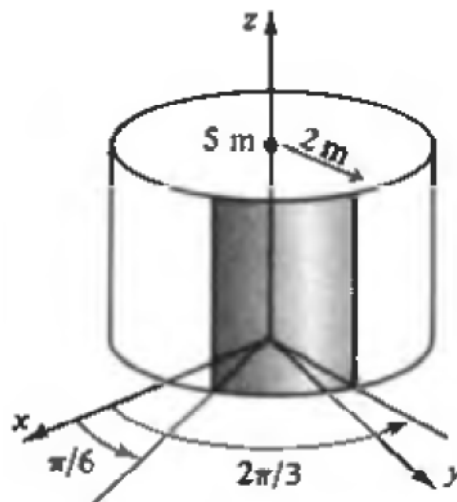
- 2] Determine the area  $S$  of a surface in a spherical coordinate system as  $r = 2\text{ m}$  and  $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{3}$  rad .

$$\left[ A = 5.205 \text{ m}^2 \right]$$

- 3] Use the spherical coordinate system to find the area of the strip  $\alpha \leq \theta \leq \beta$  on the spherical shell of  $r = a$  show this strip by sketching. What result when  $\alpha = 0$  and  $\beta = \pi$  ?

$$\left[ \begin{array}{l} A = 2\pi a^2 [\cos \alpha - \cos \beta] \\ A = 4\pi a^2 \end{array} \right]$$

- 4] Use the cylindrical coordinate system to find the area of the curved surface of a right circular cylinder where:  $r = 2\text{ m}$  ,  $h = 5\text{ m}$  and  $30^\circ \leq \phi \leq 120^\circ$  as shown in fig



$$\left[ A = 5\pi \text{ m}^2 \right]$$

5] Given the point  $P$  (5m,  $60^\circ$ , 2m) and  $Q$  (2m,  $110^\circ$ , -1m)

(a) Find the distance  $R_{PQ}$

(b) Give a unit vector in Cartesian coordinates at  $P$  that is directed towards  $Q$

(c) Give a unit vector in cylindrical coordinates at  $P$  that is directed towards  $Q$

$$\left[ \begin{array}{l} R_{PQ} = 5.014 \text{ m} \\ \bar{a}_{PQ} = -0.635\bar{a}_x - 0.489\bar{a}_y - 0.598\bar{a}_z \\ \text{At point } P, \bar{a}_{PQ_{cyl}} = -0.741\bar{a}_\rho + 0.306\bar{a}_\phi - 0.598\bar{a}_z \end{array} \right]$$

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6]

(a) Find  $\bar{a}_x$  in the spherical components at  $P$  (3, -4, 5)

(b) Find  $\bar{a}_\theta$  in Cartesian components at  $P$

$$\left[ \begin{array}{l} \bar{a}_x = 0.424\bar{a}_r + 0.424\bar{a}_\theta + 0.8\bar{a}_\phi \\ \bar{a}_\theta = 0.424\bar{a}_x - 0.565\bar{a}_y - 0.707\bar{a}_z \end{array} \right]$$

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7] A closed surface is defined in spherical coordinates by  $3 \leq r \leq 5$ ,

$$0.1\pi \leq \theta \leq 0.3\pi, \quad 1.2\pi \leq \phi \leq 1.6\pi$$

(a) Find the volume enclosed.

(b) Find the total surface area.

$$\left[ \begin{array}{l} V = 14.912 \\ \text{Total Area} = 36.8125 \end{array} \right]$$

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8] Transform  $\bar{A} = y\bar{a}_x + x\bar{a}_y + \frac{x^2}{\sqrt{x^2 + y^2}}\bar{a}_z$  to cylindrical coordinates.

$$[\bar{A}_{cyl} = \rho \sin 2\phi \bar{a}_\rho + \rho \cos 2\phi \bar{a}_\phi + \rho \cos^2 \phi \bar{a}_z]$$